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Controller Reduction by the Block Balanced Realization with Frequency Weight

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Abstract

In this paper, we propose an effective frequency weight for -controller reduction using the block balanced realization method.

To decrease the closed-loop performance degradation, we need to take account the closed-loop configuration in controller reduction process. The block balanced realization method was developed to consider the closed-loop configuration. In our method, we employ the block balanced realization method to reduce the order of -controller. Moreover to decrease the closed-loop performance degradation, we use the scaling matrix D as frequency weight.

A numerical example is presented to illustrate the effectiveness of our method.

1 Introduction

As practical realization of robust controller design method such as H_∞ control or -design progressed, controller reduction is becoming an important problem. When we use these robust controller design method, a high-order controller is provided generally. A low-order controller is desirable from a standpoint of cost and reliability. Thus when we use controller actually controller reduction is necessary.

Previously the controller reduction applied existing model reduction methods directly. However many of those methods focus on input-output characteristics of approximated model. Hence when closed-loop system include approximation model may yield problems such as decreased stability or performance degradation. To solve this problem, Enns proposed introduction of a frequency weight into the balanced realization method[?]. In addition, several frequency weights that considered characteristics of closed-loop system were also proposed[?]. These methods deal with model reduction problem of open-loop system basically.

On the other hand, to consider closed-loop configuration the block balanced realization method was proposed. This method performs controller reduction that considered input-output characteristics of closed-loop system including controlled object and controller.

The block balanced realization is one of the effective methods of controller reduction that saved closed-loop characteristic. When append a frequency weight to the block balanced realization method, controller reduction may be more effective. However, an optimal frequency weights for -controller reduction are not yet found.

Therefore, in this paper, an effective frequency weight for -controller reduction using the block balanced realization method is proposed. First, algorithm of the block balanced realization is described. Next, an effective frequency for -controller reduction is presented. Primary objective of this paper is to propose a frequency weight that decreases the closed-loop performance degradation. The effectiveness of our proposed method is confirmed by simulation.

2 Method

2.1 The block balanced realization

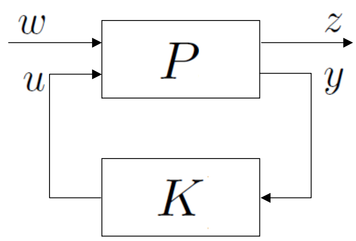

Figure 1: The closed-loop system

Consider the closed-loop system shown in Figure

1, with external input w, controlled output z, control input u, and measured output y.

Let, a generalized plant P and a controller K are

$$P = \begin{matrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{matrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right].$$

$$K = \begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array},$$

where the order of K is n. Using LFTs, transfer function from w to z is given by

$$T_{zw} = F_l(P,K) = P_{11} + P_{12}K(I \quad P_{22}K)^{-1}P_{21}$$
$$= \left[\begin{array}{cc|c} A + B_2LD_kC_2 & B_2LC_k & B_1 + B_2LD_kD_{21} \\ B_kFC_2 & A_k + B_kFD_{22}C_k & B_kFD_{21} \\ \hline C_1 + D_{12}D_kFC_2 & D_{12}LC_k & D_{11} + D_{12}D_kFD_{21} \end{array}\right].$$

where $L = (I \quad D_kD_{22})^{-1}$ and $F = (I \quad D_{22}D_k)^{-1}$.

Let,input frequency weight W_i and output frequency weight W_o are

$$W_i = \begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}, \quad W_o = \begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}.$$

Then

$$W_oGW_i = \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & \bar{D} \end{array}\right]$$
$$= \left[\begin{array}{cc|c} \bar{A}_{11} & \bar{A}_{12}C_k & \bar{B}_1 \\ B_k\bar{A}_{21} & A_k + B_kFD_{22}C_k & B_kFD_{21}D_i \\ \hline \bar{C}_1 & D_oD_{12}LC_k & \bar{D} \end{array}\right]. \tag{1}$$

For the system W_oGW_i, solve two lyapnov equations:

$$\bar{A}U + U\bar{A}^T + \bar{B}\bar{B}^T = 0$$
$$\bar{A}^TY + Y\bar{A} + \bar{C}^T\bar{C} = 0.$$

U and Y are the controllability and observability Gramians of W_oGW_i, and are partitioned compatibly with $\bar{A}$ in (1) as

$$U = \begin{matrix} U_1 & U_{12} \\ U_{12}^T & U_2 \end{matrix}, \quad Y = \begin{matrix} Y_1 & Y_{12} \\ Y_{12}^T & Y_2 \end{matrix}.$$

Then there exists a nonsingular matrix T such as that

$$TU_2T^T = (T^{-1})^TY_2T^{-1} = diag(\sigma_1, \quad ,\sigma_n) = \Sigma$$

where $\sigma_i \quad \sigma_{i+1} \quad 0$.

Transform and partition K as

$$K = \begin{array}{c|c} TA_kT^{-1} & TB_k \\ \hline C_kT^{-1} & D_k \end{array} = \left[\begin{array}{cc|c} A_r & A_{k12} & B_r \\ A_{k21} & A_{k22} & B_{k2} \\ \hline C_r & C_{k2} & D_k \end{array}\right].$$

Finally, the low-order controller K_r is obtained by truncation:

$$K_r = \begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}.$$

2.2 Proposed method algorithm

We obtain low-order controller by the following five procedures.

Step1: We derive the -controller K and scaling matrix D_r^{-1}, D_l by use of D-K iteration from a controlled object G and a structured uncertainty Δ.(Figure 2)

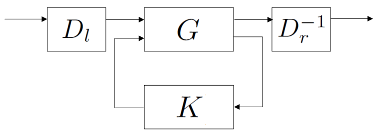

Figure 2: Derivation of K by D-K iteration

Step2: Configure a generalized plant P from a controlled object G and obtained scaling matrixes D_l, D_r^{-1}.($P = D_lGD_r^{-1}$/Figure 3)

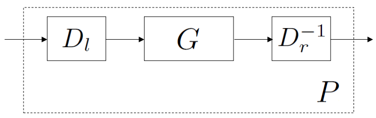

Figure 3: A generalized plant

Step3: Perform -analysis by use of the closed-loop system $F_l(P,K)$ of a generalized plant P and the -controller K, and derive scaling matrixes D_{l2}, D_{r2}^{-1} again.(Figure 4)

Step4: Perform the block balanced realization to controller K by using the scaling matrixes D_{l2}, D_{r2}^{-1} as frequency weight W_i, W_o.

Step5: The low-order controller K_r is obtained by truncation. Further make the closed-loop system

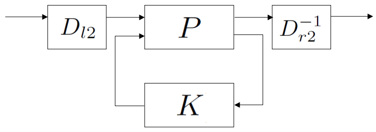

Figure 4: Derivation of scaling matrixes D by the μ-analysis

$F_l(P, K_r)$ from a generalized plant P and obtained low-order controller K_r.(Figure 5)

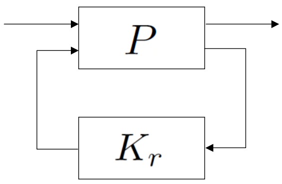

Figure 5: The closed-loop system using a reduction controller

3 A numerical example

As a numerical example, we treat combat plane model "HIMAT". A controlled object of this system is 10'th order, 6 outputs, 8 inputs, and state-space realization is as follows. We executed D-K iteration three times about this system. Table 1 shows H_∞ norm of closed loop system and the order of obtained scaling matrix D_r, D_l and controller K. A generalized plant P is equivalent to a controlled object G from Table 1, when iteration of the first time. The second time or later, we generate a generalized plant P from a controlled object G and matrixes D.

$$A = \begin{bmatrix} 0.023 & 37 & 19 & 32 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.9 & 0.98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.012 & 12 & 2.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 0 & 0.018 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54 & 0 & 0 & 0 & 0.018 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 320 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 320 \end{bmatrix}.$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.41 & 0 & 0 & 0 & 0 & 0 \\ 78 & 22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.94 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.94 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0.41 & 0 \\ 78 & 22 \\ 0 & 0 \\ 700 & 0 \\ 0 & 700 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 700 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 700 & 0 & 0 & 0 & 0 \\ 0 & 29 & 0 & 0 & 0 & 0 & 0.94 & 0 & 0 & 0 \\ 0 & 0 & 0 & 29 & 0 & 0 & 0 & 0.94 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 57 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 57 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix}.$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix}, D_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 1: Resultant K and D from D-K iteration

Iteration Number	1	2	3
Controller Order	10	30	30
D matrix Order(*1)	-	10	10
$F_l(P, K)$	2.0361	1.0755	0.96491

*1 Matrix 'D' is equivalent with $D_l, D_r{}^{1}$ of "Step2"

We performed reduction of controller as per three methods of 'Proposed method ', 'BT 'and the 'BBT'. Where, 'Proposed method' represent 'The block balanced realization with frequency weight'. 'BT' represent 'The balanced realization', and,'BBT' represent 'The block balanced realization without frequency'. Table 2 ∼ 4 summarizes the result of H_∞ norm of the closed-loop system $\|F_l(P, K_r)\|_\infty$ that made by an obtained low-order controller K_r and a generalized plant P.

We see from Table 2 ∼ 4 that the controller was reduced to stability until 4'th order by all methods. We focus on 4'th order of Table 2, H_∞ norm of a proposed method was appreciably lower values than 'BBT' method. In addition, we see from Table 3∼4 that the H_∞ norm was lower values in comparison with conventional method. Therefore we found from the result that proposed method had an effect on controller reduction of the -controller.

4 Conclusions

In this paper, we focus on controller reduction problem of the -controller, we proposed an effective frequency weight for -controller reduction using the block balanced realization method. In addition, the effectiveness of our proposed method was confirmed by simulation. As a result, performance degradation was improved than conventional method. In conclusion, the blocked balanced realization method that employed the scaling matrix D as frequency weight improved performance degradation by -controller reduction. However, proposed method did not produce dramatic improvement than 'BT' method. Moreover proposed method was unstable by a number of orders. We will think that have need to continue study about more an effective frequency weight.

References

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Table 2: $\|F_l(P, K_r)\|_\infty$ at Iteration Number=1

Order of K_r	Proposed method	BT	BBT
9	2.0361	2.0490	2.0361
8	2.0432	2.0499	2.0408
7	2.0413	2.0500	2.0489
6	2.2666	2.0607	4.1688
5	4.1129	3.8943	5.6221
4	5.4113	5.6100	14.204
3	unstable	unstable	unstable
2	unstable	unstable	unstable
1	unstable	unstable	unstable

Table 3: $\|F_l(P, K_r)\|_\infty$ at Iteration Number=2

Order of K_r	Proposed method	BT	BBT
20	1.0755	1.0755	1.0755
15	1.0797	1.0799	1.0788
10	1.1315	1.2114	1.1085
9	1.1166	1.1520	1.1115
8	1.4575	1.1537	unstable
7	1.1906	1.1542	1.1523
6	1.2781	1.7449	1.2669
5	1.6863	1.4433	1.8256
4	2.7053	2.7496	2.6569
3	unstable	11.221	unstable
2	unstable	unstable	unstable
1	unstable	unstable	unstable

Table 4: $\|F_l(P, K_r)\|_\infty$ at Iteration Number=3

Order of K_r	Proposed method	BT	BBT
20	0.9648	0.9645	0.9647
15	0.9673	0.9696	0.9673
10	0.9706	1.0382	0.9722
9	0.9885	1.0381	1.0364
8	1.2015	1.3824	1.3480
7	unstable	1.8060	1.2359
6	1.2004	1.1822	1.2974
5	1.5545	1.9111	1.7744
4	3.6131	unstable	4.6071
3	unstable	unstable	unstable
2	unstable	unstable	unstable
1	unstable	unstable	unstable